(Non-)perturbative tests of the AdS/CFT correspondence

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I summarize perturbative and non-perturbative field theory tests of the holographic correspondence between type IIB superstring on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM theory. The holographic duality between D-instantons and YM instantons is briefly described. Non renormalization properties of two- and three-point functions of CPO's and their extremal and next-to-extremal correlators are then reviewed. Finally, partial non-renormalization of four-point functions of lowest CPO's is analyzed in view of the interpretation of short distance logarithmic behaviours in terms of anomalous dimensions of unprotected operators.

The AdS/CFT correspondence [1] is an unprecedented tool in the study of the interplay between gauge theory and gravity. In the simplest case it relates type IIB superstring on $AdS_5 \times S^5$ to $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) with gauge group $SU(N)$. The correspondence is "holographic" in that the gauge theory lives on the boundary of AdS. The conjecture is motivated by the study of the low-energy dynamics of D-branes [2]. Their open strings excitations include massless vector supermultiplets and a stack of N coincident D3-branes is governed by $U(N)$ $\mathcal{N} = 4$ SYM in $D = 4$. The remarkable fact about this gauge theory is its exact superconformal invariance. This reflects into the the dilaton being constant and the metric being nowhere singular for the D3-brane solution! The D3-brane can thus be viewed as a smooth soliton interpolating between maximally supersymmetric flat Minkowski spacetime at infinity and maximally supersymmetric $AdS_5 \times S^5$ near the horizon. The AdS scale L is related to the RR 5-form flux N by $L^4 = 4\pi g_s N \alpha'^2$. Due to the red-shift, the geometry near the horizon is dual to the low-energy limit where bulk supergravity effectively decouples from the boundary dynamics. Open-closed string duality suggests a perfect equivalence between the two descriptions should take place when

$$g_s = \langle e^{\Phi} \rangle = \frac{g^2_{YM}}{4\pi} \qquad \vartheta_s = \langle \chi \rangle = \frac{\theta}{2\pi} \tag{1}$$

and N is identified with the number of colours. The type IIB "partition function" plays the role of a generating functional

$$Z_{IIB}[\Phi[J]] = Z_{SYM}[J] \ . \tag{2}$$

The boundary data $J(x)$ for type IIB bulk fields $\Phi_M(x, \rho)$ are viewed as sources for gauge-invariant SYM local composite operators $\mathcal{O}_\Delta(x)$. Since string theory on AdS spaces with RR background is still poorly understood even at the classical level, it is necessary to perform a double expansion in powers of the string coupling g_s and inverse tension α' or, equivalently, in powers of $1/N$ and $1/\kappa$, where $\kappa = g^2_{YM} N$ is the reduced 't Hooft coupling. Taking N to infinity one is effectively neglecting string loops and restricting to the tree level (sphere) dual to the "planar series". This is still insufficient for computational purposes. In order to suppress higher derivative corrections one has to restrict to large κ *i.e.* to the strong coupling regime in the dual gauge theory. In principle, one can systematically take into account worldsheet corrections, string-loops and non-perturbative D-instanton corrections [3].

A string description of confinement has been long sought for. Just before Maldacena's pro-

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posal, Alexander Polyakov observed that one of the drawbacks of previous attempts, *i.e.* the lack of *zig-zag* symmetry of the non-critical string, could be cured by assuming the flow to a fixed point at vanishing Liouville field $\rho = 0$ [4]. Maldacena's proposal then looks like what the Doctor orders in that it puts forward the existence of a fifth coordinate ρ (transverse to the boundary) that could be identified with the Lioville mode or, possibly equivalently, with a renormalization scale. What sounds surprising is that $\mathcal{N} = 4$ SYM theory is not confining and has no mass-gap in the superconformal phase.

The superisometry group of $AdS_5 \times S_5$, $SU(2,2|4)$, acts by superconformal transformations on the boundary CFT. Unitary irreducible representations of $PSU(2,2|4)$ are labelled by the quantum numbers $\{\Delta, j_L, j_R; [k,l,m]\}$. Δ is the scaling dimension, (j_L, j_R) denote the $SU(2)_L \times SU(2)_R$ spins, and $[k,l,m]$ are the Dynkin labels of an irrep $\mathbf{r}$ of the $SU(4)$ R-symmetry group. The fundamental SYM fields $\{\varphi^i, \lambda^A, F_{\mu\nu}\}$ describing the lowest lying open-string excitations belong to the singleton representation and live on the boundary. Gauge invariant composite operators $\mathcal{O}_\Delta$ dual to type IIB bulk fields Φ_M form doubleton or long multiplets. In particular, the "ultra-short" $\mathcal{N} = 4$ supercurrent multiplet is dual to the "massless" AdS supergravity multiplet. Operators dual to higher KK excitations assemble into short multiplets with spin $j = j_L + j_R$ ranging from -2 to 2 and Dynkin labels $k, m \leq 2$. The lowest component is a chiral primary operators (CPO)

$$\mathcal{Q}^{(i_1 i_2 \ldots i_\ell)}_{[0,\ell,0]} = \sum_\sigma Tr(\varphi^{\sigma(i_1)} \varphi^{\sigma(i_2)} \ldots \varphi^{\sigma(i_\ell)} - traces)$$

of dimension $\Delta = \ell$ belonging to the ℓ-fold traceless symmetric product of the $\mathbf{6}$ of $SU(4) \approx SO(6)$. Other shortenings are possible for multi-trace operators. For scalar primaries, for instance, shortening occurs when $\mathbf{r} = [k,\ell,k]$ and $\Delta = \ell + 2k$ or $\mathbf{r} = [k+2n,\ell,k]$ and $\Delta = \ell + 2k + 3n$ [5]. The spin ranges over 6 or 7 units respectively. Operators dual to string excitations with AdS masses of order $1/\sqrt{\alpha'}$ belong to long multiplets. Their spin ranges over 8 units and their dimension is expected to grow like $\Delta \approx \kappa^{1/4}$ in the strong coupling limit. One such example is the $\mathcal{N} = 4$ Konishi multiplet whose lowest component is the scalar $SU(4)$ singlet

$$\mathcal{K}_{\mathbf{1}} = Tr(\varphi^i \varphi_i) \tag{3}$$

with naive dimension $\Delta = 2$.

In addition to perturbative symmetries, both type IIB superstring and $\mathcal{N} = 4$ SYM are expected to be invariant under S-duality. Charge quantization of solitonic states breaks the non-compact $SL(2,R)$ group of "classical" type IIB supergravity to $SL(2,Z)$. Strings, 5-branes and 7-branes form multiplets of this discrete non-compact symmetry while the D3-brane is invariant. Similarly stable dyonic states of $\mathcal{N} = 4$ SYM transform into one another under $SL(2,Z)$. For simply laced gauge groups the theory is expected to be self-dual since the spectra of electric and magnetic charges coincide. For non-simply laced groups S-duality maps one into the other. Theories with orthogonal and symplectic groups are dual to the near horizon geometry of D3-brane configurations in the presence of un-orientifold planes with quantized possibly vanishing two form background [6,7]. The breaking of $SL(2,R)$ to $SL(2,Z)$ is manifest in the D-istanton corrections to the type IIB effective action [3]. As we will see these are dual to SYM instanton corrections [8,7,9].

Finally, three $U(1)$'s play an interesting subtle role in the correspondence. The first, $U(1)_Z$, is a central extension of $PSU(2,2|4)$. Fundamental fields as well as their composites are neutral with respect to it so that one usually neglects it. It is conceivable that solitonic states could carry non-vanishing $U(1)_Z$ charge and form novel $SU(2,2|4)$ multiplets [10]. The second, $U(1)_C$, is the abelian factor in $U(N)$. ¿From the D3-brane perspective it corresponds to the center of mass degrees of freedom. Its low-energy dynamics on the boundary cannot be reproduced by the bulk supergravity action. In the supergravity limit one would like to say there is an additional singleton multiplet not captured by the correspondence if not for its contribution to "boundary anomalies" [11]. When higher-derivative Born-Infeld corrections take over the theory is better described in terms of open strings. The third, $U(1)_B$, is a "bonus" symmetry of a restricted class of correlation functions and their dual amplitudes [12]. In SYM it corresponds to a chiral rotation accompanied by a continuous electric-magnetic duality transformation. Its type IIB counterpart is the $U(1)_B$ anomalous chiral symmetry. When supergravity loops and higher derivative string corrections are negligible the "bonus" symmetry becomes a true symmetry. Independently of the coupling κ and N, all 2-point correlation functions, 3-point functions with at most one insertion of unprotected operators and 4-point functions of single-trace protected operators seem to respect this symmetry [12].

In a superconformal field theory, two-point functions of normalized primary operators $\mathcal{O}_\Delta$ are completely specified by their dimensions

$$\langle \mathcal{O}^\dagger_\Delta(x)\mathcal{O}_\Delta(y)\rangle = \frac{1}{(x-y)^{2\Delta}} \quad (4)$$

The first step in computing two-point functions of (scalar) gauge-invariant composite operators using the correspondence is to solve the linearized field equation [13–15]

$$-\nabla^2\Phi_M + M^2\Phi_M = 0 \quad (5)$$

with near-boundary behaviour[2]

$$\Phi_M(\rho,x) \to \rho^{4-\Delta}J(x) \quad (6)$$

as $\rho \to 0$. The solution may be expressed in terms of the bulk-to-boundary propagator

$$K_\Delta(\rho,x;x') = \frac{a_\Delta \rho^\Delta}{(\rho^2+(x-x')^2)^\Delta} \quad (7)$$

It is not at all a coincidence that K_Δ resembles a YM instanton form factor. Plugging K_Δ into the scalar field equation one finds the following mass-to-dimension relation

$$(ML)^2 = \Delta(\Delta-4) \quad (8)$$

and its inverse

$$\Delta = 2 \pm \sqrt{4+(ML)^2} \quad (9)$$

[2]The other possible near-boundary behaviour $\Phi_M(\rho,x) \to \rho^\Delta V(x)$ corresponds to turning on a VEV $\langle\mathcal{O}\rangle = V$ for the operator dual to Φ.

Δ is real, as expected in unitary theory, once the Breitenlohner Freedman bound $(ML)^2 \leq -4$ is enforced [16]. Only the positive branch is relevant for $\mathcal{N}=4$ SYM. Carefully computing the quadratic on-shell type IIB action and differentiating wrt to the sources J exactly reproduce the field theory result (4).

Three-point functions, although largely fixed by superconformal invariance, encode the dynamics of the theory since one can in principle reconstruct all correlation functions by factorization. A particularly interesting class of three-point functions are those of CPO's

$$\langle Q_{\ell_1}(x_1)Q_{\ell_2}(x_2)Q_{\ell_3}(x_3)\rangle = \frac{C(\ell_1,\ell_2,\ell_3)}{\prod (x^2_{ij})^{\ell_i+\ell_j-\frac{1}{2}\Sigma}} \quad (10)$$

where $x_{ij} = x_i - x_j$ and $\Sigma = \ell_1+\ell_2+\ell_3$. The trilinear couplings $C(\ell_1,\ell_2,\ell_3)$ can be easily computed at weak coupling for large N. In order to perform the dual AdS computation one has to go beyond the linearized approximation. Quadratic terms in the field equations, or equivalently cubic terms in the action are necessary. This is in general very complicated but the computation turns out to be feasible for CPO's. Quite remarkably, one finds the same result as in free-field theory at large N [17]. The exact matching suggests the validity of a non-renormalization theorem for any κ and N. This has been tested at one-loop [18] and to two-loops [19,20]. A judicious usage of the "bonus" $U(1)_B$ symmetry [12] in the context of $\mathcal{N}$=2 harmonic superspace gives a demonstration of the non-renormalisation of two- and three-point functions of CPO's [21]. The extremal case, $\ell_1 = \ell_2+\ell_3$, is subtler. We will return to this issue after discussing non-perturbative effects.

Using the AdS/CFT dictionary (1), the charge-k type IIB D-instanton action coincides with the action of a charge-k YM instanton. This strongly indicates a correspondence between these sources of non-perturbative effects [8,7]. Moreover, it is known that the $k=1$ $SU(2)$ YM instanton moduli space coincides with Euclidean AdS_5 and the same is true for a type IIB D-instanton on AdS_5. The fifth radial coordinate transverse to the boundary plays the role of the YM instanton size ρ. The correspondence can be made more quantitative by noticing that the super-instanton measure contains an overall factor g^8_{YM} that arises from the combination of bosonic and fermionic zero-mode norms and exactly matches the power expected on the basis of the AdS/CFT correspondence.

The computation of the one-instanton contribution to the SYM correlation function $G_{16} = \langle\Lambda(x_1)\ldots\Lambda(x_{16})\rangle$, where $\Lambda^A = Tr(F_{\mu\nu}\sigma^{\mu\nu}\lambda^A)$, is the fermionic composite operator dual to the type IIB dilatino, and its comparison with the D-instanton contribution to the dual type IIB amplitude has given the first truly dynamical test of the correspondence [9]. Correlation functions of this kind are almost completely determined by the systematics of fermionic zero-modes in the YM instanton background. Performing (broken) superconformal transformations on the instanton field-strength

$$F_{\mu\nu} = K_2(\rho, x; x')\sigma_{\mu\nu} \tag{11}$$

yields the relevant gaugino zero-modes

$$\lambda^A = \frac{1}{2}F_{\mu\nu}\sigma^{\mu\nu}\zeta^A \tag{12}$$

where $\zeta^A = \eta^A + x\cdot\sigma\bar{\xi}^A$, with η^A and $\bar{\xi}^A$ constant Weyl spinors of opposite chirality. The exact matching with the corresponding type IIB amplitude is quite impressive and somewhat surprising. Indeed the SYM computation initially performed for an $SU(2)$ gauge group at weak coupling [9], *i.e.* in a regime which is clearly far from the large N limit at strong coupling, has since then been extended to the $k = 1$ instanton sector for $SU(N)$ and to any k in the large N limit [22]. The resulting 16-point functions have the same dependence on the insertion points. In the large N limit the overall coefficients of the dominant terms are those predicted by the analysis of type IIB D-instanton effects [3]. For $N \neq 2$ all but the 16 superconformal zero-modes (12) are lifted by Yukawa interactions. Additional bosonic coordinates parameterizing S^5 appear in the large N limit as bilinears in the lifted fermionic zero-modes.

Other correlators that are related by supersymmetry to the Λ^{16} function and can thus saturate the 16 exact zero-modes have been computed [9,22,23]. Correlators that cannot absorb the exact zero-modes receive vanishing contributions. This is the case for two- and three-point functions of CPO's as well as for extremal and next-to-extremal correlators to which we now turn our attention.

The correlator of CPO's

$$G_{extr} = \langle\mathcal{Q}^{(\ell)}(x)\mathcal{Q}^{(\ell_1)}(x_1)\ldots\mathcal{Q}^{(\ell_n)}(x_n)\rangle \tag{13}$$

is said to be "extremal" when $\ell = \ell_1+\ell_2+\ldots+\ell_n$. It is easy to check that (13) contains only one $SU(4)$ tensor structure so that computing (13) is equivalent to computing

$$\langle Tr[(\phi)^\ell(x)]Tr[(\phi^\dagger)^{\ell_1}(x_1)]\ldots Tr[(\phi^\dagger)^{\ell_n}(x_n)]\rangle \tag{14}$$

where ϕ is $\phi^I = \varphi^I + i\varphi^{I+3}$, with, say, $I = 1$. The tree-level contribution corresponds to a diagram with ℓ lines exiting from the point x, which form n different "rainbows" connecting x to the points x_i, the ith rainbow containing ℓ_i lines. The result is schematically of the form

$$G(x, x_1, \ldots, x_n) = c(g, N)\prod_i (x - x_i)^{-2\ell_i} \ . \tag{15}$$

The dual supergravity computation is very subtle in that the relevant AdS integrals are divergent but at the same time extremal trilinear coupling are formally vanishing [17]. If one carefully analytically continue the computation away from extremality, one finds a non-vanishing result of the same form as at the tree-level in SYM theory [24]. One is thus lead to conjecture extremal correlators should satisfy a non-renormalization theorem of the same kind as two- and three-point functions of CPO's. This has been tested both at one-loop and non-perturbatively [25].

At one loop, there are two sources of potential corrections. The first corresponds to the insertion of a vector lines connecting the chiral lines of the same rainbow. Its vanishing is in some sense related to the vanishing of one-loop corrections to two-point functions of CPO's. The second corresponds to insertion of vector lines connecting lines belonging to different rainbows. Its vanishing is in the same sense as above related to the vanishing of one-loop correction to 3-point functions of CPO's. The same analysis can be repeated step by step in the case of extremal correlators involving multi-trace operators in short multiplets [25].

As far as the instanton contributions are concerned, it is easy to check that (14) cannot absorb the relevant 16 zero-modes. The induced scalar zero-modes read

$$\varphi^i = \frac{1}{2}\tau^i_{AB}\zeta^A F_{\mu\nu}\sigma^{\mu\nu}\zeta^B \tag{16}$$

and the 4 exact zero-modes with flavour $I = 1$ could only be possibly absorbed at x. Since however $\zeta(x)^4 = 0$, the non perturbative corrections to (14) vanishes for any instanton number and for any gauge group in the leading semiclassical approximation.

Other correlators, involving only one $SU(4)$ singlet projection, enjoy similar non-renormalization properties. Two- and three-point functions of CPO's belong to this class. The identification of $U(1)_B$-violating nilpotent super-invariants beginning at five points [26] prevents one from generically extending the same argument to higher-point functions. However the absence of the relevant nilpotent super-invariants for next-to-extremal correlators with $\ell = (\sum_i \ell_i)-2$ allows one to add them to the above list [26]. The absence of one-loop and instanton corrections in this case [27] can be verified along the same line as for the extremal ones [25]. Supergravity computations confirm the weak coupling result [28] and suggest that near-extremal correlators, with $\ell = (\sum_i \ell_i) - 4$, satisfy a sort of "partial" non renormalization. The *a priori* independent contributions to a given correlation function are functionally related to one another. Functional relations of this form easily emerge in instanton computations [29]. Some additional effort allows one to derive them in perturbation theory [30,19,29].

The dynamics of the theory is elegantly encoded in the four-point functions. The simplest ones have been computed both at weak coupling, up to order g^4 [29,20,30,19] as well as in the semiclassical instanton approximation [9], and at strong coupling from the AdS perspective [31,32]. At short distance they generically display logarithmic behaviours that are to be interpreted in terms of anomalous dimensions. At first sight this might seem surprising in a theory, such as $\mathcal{N} = 4$ SYM, that is known to be finite. Indeed, operators which belong to short multiplets have protected scaling dimensions and cannot contribute to the logarithmic behaviours. Completeness of the operator product expansion (OPE) requires the inclusion of "unprotected" operators in addition to the "protected" ones. Single-trace operators in Konishi-like multiplets [5,33] contribute to the logarithms at weak coupling but are expected to decouple at strong coupling. On the contrary unprotected multi-trace operators that are holographically dual to multi-particle states appear both at weak and at strong coupling since their anomalous dimensions are at most of order $1/N^2$ [2,31,34,29,20,35].

To clarify the point in a simpler setting, consider the two-point function of a primary operator of scale dimension $\Delta = \Delta^{^{(0)}} + \gamma$. In perturbation theory $\gamma = \gamma(g_{_{YM}})$ is expected to be small and to admit an expansion in the coupling constant $g_{_{YM}}$. Expanding in γ yields

$$\langle \mathcal{O}^\dagger_\Delta(x)\mathcal{O}_\Delta(y)\rangle = \frac{a_\Delta}{(x-y)^{2\Delta^{(0)}}} \times \tag{17}$$
$$\left(1 - \gamma\log[\mu^2(x-y)^2] + \frac{\gamma^2}{2}(\log[\mu^2(x-y)^2])^2 + \ldots\right) .$$

Although the exact expression (4) given above is conformally invariant, at each order in γ (or in $g_{_{YM}}$) (18) contains logarithms that are an artifact of the pertrubative expansion.

Similar considerations apply to arbitrary correlation functions. Assuming the convergence of the OPE, a four-point function of primary operators can be schematically expanded as

$$\langle \mathcal{Q}_A(x)\mathcal{Q}_B(y)\mathcal{Q}_C(z)\mathcal{Q}_D(w)\rangle = \tag{18}$$
$$\sum_K \frac{{C_{AB}}^K(x-y,\partial_y)}{(x-y)^{\Delta_A+\Delta_B-\Delta_K}}\frac{{C_{CD}}^K(z-w,\partial_w)}{(z-w)^{\Delta_C+\Delta_D-\Delta_K}}$$
$$\langle \mathcal{O}_K(y)\mathcal{O}_K(w)\rangle ,$$

where K runs over a (possibly infinite) complete set of primary operators. Descendants are implicitly taken into account by the presence of derivatives in the Wilson coefficients, C's. To simplify formulae we assume that $\mathcal{Q}$'s are protected operators, *i.e.* they have vanishing anomalous dimensions. In general the operators $\mathcal{O}_K$ may have anomalous dimensions, γ_K, so that

$\Delta_K = \Delta_K^{(0)} + \gamma_K$. Similarly $C_{IJ}{}^K = C_{IJ}^{(0)}{}^K + \eta_{IJ}{}^K$. Indeed, although three-point functions of single-trace CPO's are not renormalised beyond tree level [18], *a priori* nothing can be said concerning corrections to three-point functions also involving unprotected operators.

Neglecting descendants and keeping the lowest order terms in γ and η

$$\begin{aligned} &\langle \mathcal{Q}_A(x)\mathcal{Q}_B(y)\mathcal{Q}_C(z)\mathcal{Q}_D(w)\rangle_{(1)} = \\ &\sum_K \frac{\langle \mathcal{O}_K(y)\mathcal{O}_K(w)\rangle_{(0)}}{(x-y)^{\Delta_A+\Delta_B-\Delta_K^{(0)}}(z-w)^{\Delta_C+\Delta_D-\Delta_K^{(0)}}} \times \\ &\Bigg[\eta_{AB}{}^K C_{CD}^{(0)}{}^K + C_{AB}^{(0)}{}^K \eta_{CD}{}^K + \\ &\frac{\gamma_K}{2} C_{AB}^{(0)}{}^K C_{CD}^{(0)}{}^K \log\frac{(x-y)^2(z-w)^2}{(y-w)^4}\Bigg] . \end{aligned} \tag{19}$$

whence one can extract both corrections to OPE coefficients and anomalous dimensions.

For definiteness, let us consider the four-point function of the lowest CPO's in the $\mathcal{N}$=4 current multiplet[3], defined by

$$\mathcal{Q}_{\mathbf{20}}^{ij} = \mathrm{tr}(\varphi^i\varphi^j - \frac{\delta^{ij}}{6}\varphi_k\varphi^k) . \tag{20}$$

Due to the lack of a manifestly $\mathcal{N}=4$ off-shell superfield formalism, perturbative computations have to be either performed in components or in one of the two available off-shell superfield formalisms. Although the number of diagrams is typically larger in the $\mathcal{N}=1$ superfield approach [29,20,35] its simplicity makes it more accessible than the less familiar $\mathcal{N}=2$ harmonic superpace [30,19]. It is remarkable that up to some overall normalization factors depending on the (not always standard) conventions adopted the two results are in perfect quantitative agreement with one another and in qualitative agreement with the AdS predictions at strong coupling.

Instead of computing the most general 4-point function of lowest CPO's we simply display the one-loop result for illustrative purposes. One of the six $SU(4)$ singlet projections reads

$$\begin{aligned} &G_H(x_1,x_1,x_3,x_4) = \\ &\langle(\phi^1)^2(x_1)(\phi_1^\dagger)^2(x_2)(\phi^2)^2(x_3)(\phi_2^\dagger)^2(x_4)\rangle = \\ &-\frac{2g_{YM}^2 N(N^2-1)\pi^2}{(2\pi)^{12}x_{12}^2x_{34}^2x_{13}^2x_{24}^2}\, B(r,s) , \end{aligned} \tag{21}$$

where $B(r,s)$ is a box-type integral that can be expressed as a combination of logarithms and dilogarithms as follows

$$B(r,s) = \frac{1}{\sqrt{p}}\left\{\ln(r)\ln(s) - \left[\ln\left(\frac{r+s-1-\sqrt{p}}{2}\right)\right]^2 + \right.$$
$$\left. -2\mathrm{Li}_2\left(\frac{2}{1+r-s+\sqrt{p}}\right) - 2\mathrm{Li}_2\left(\frac{2}{1-r+s+\sqrt{p}}\right)\right\} .$$

As indicated, $B(r,s)$ depends only on the two independent conformally invariant cross ratios

$$r = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2} , \quad s = \frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2} . \tag{22}$$

and

$$p = 1 + r^2 + s^2 - 2r - 2s - 2rs . \tag{23}$$

The other *a priori* independent 4-point function

$$\begin{aligned} &G_V(x_1,x_1,x_3,x_4) = \\ &\langle(\phi^1)^2(x_1)(\phi_1^\dagger)^2(x_2)(\phi^1)^2(x_3)(\phi_1^\dagger)^2(x_4)\rangle \end{aligned} \tag{24}$$

The non-perturbative contributions, computed in [9,29], are quite involved and we refrain to display them. We simply notice that the relation

$$\begin{aligned} &(x_1-x_3)^2(x_2-x_4)^2 G_H(x_1,x_1,x_3,x_4) = \\ &(x_1-x_4)^2(x_2-x_3)^2 G_V(x_1,x_1,x_3,x_4) \end{aligned}$$

can be easily derived from the systematics of the fermionic zer-modes [29]. Some additional effort allows one to derive it in perturbation theory [30,19,29,20]. The AdS computation is even more involved and the final result is quite uninspiring [32].

In order to extract some physics one has to perform an OPE analysis [29,20,35,34,31,28]. Restricting for brevity our attention to the sectors **1**, **20′**, **84**, and **105** the results can be summarized as follows[4].

[3] AdS computations have also produced 4-point functions of the scalar $SU(4)$ singlets in the dilaton-axion sector [31] that are less amenable to explicit computations at weak coupling. To the best of my knowledge the only concrete proposal has been made in [36] for the instanton contribution.

[4] Recall that in addition to these irreps, $\mathbf{20'}\times\mathbf{20'}$ contains $\mathbf{15}+\mathbf{175}$ in the antisymmetric part.

In the **105** one finds only subdominant logarithms, consistently with the expected absence of any corrections to the dimension of protected single- and double-trace operators of dimension $\Delta = 4$ in the **105**.

In the **84** channel, the dominant contribution at one and two loops is purely logarithmic and consistent with the exchange of the operator $\mathcal{K}_{\mathbf{84}}$ in the Konishi multiplet. The absence of dominant logarithmic terms in the instanton as well as AdS results suggests confirms the absence of any corrections to the dimension and trilinear of a protected operator $\hat{\mathcal{D}}_{\mathbf{84}}$ of dimension 4, defined by subtracting the Konishi scalar $\mathcal{K}_{\mathbf{84}}$ from the projection on the **84** of the naive normal ordered product of two $\mathcal{Q}_{\mathbf{20}'}$.

In the $\mathbf{20}'$ sector, there is no dominant logarithm suggesting a vanishing anomalous dimension for the unprotected operator $:\mathcal{Q}_{\mathbf{20}'}\mathcal{Q}_{\mathbf{20}'}:_{\mathbf{20}'}$. This striking result seems to be a consequence of the partial non-renormalization of 4-point functions of CPO's that is valid not only at each order in perturbation theory (beyond tree level!) but also non-perturbatively and at strong coupling (AdS). In order to disentangle the various scalar operators of naive dimension 4 exchanged in this channel it is necessary to compute other independent 4-point functions involving the insertions of the lowest Konishi operator $\mathcal{K}_{\mathbf{1}}$ [35].

The analysis of the singlet channel is very complicated by the presence of a large number of operators. In perturbation theory one has logarithmically-dressed double pole associated to the exchange of $\mathcal{K}_{\mathbf{1}}$ with[5]

$$\gamma^{(1)}_{\mathcal{K}} = 3\frac{g^2_{YM}N}{4\pi^2} \qquad \gamma^{(2)}_{\mathcal{K}} = -3\frac{g^2_{YM}N}{16\pi^2} \tag{25}$$

Non-perturbative and strong coupling results only show a logarithmic singularity that is associated to the exchange of some double-trace unprotected operator $\mathcal{O}_{\mathbf{1}}$ with $\gamma \approx 1/N^2$.

The picture that emerges is very interesting. In addition to protected single an multi-trace operators satisfying shortening conditions as well as single- and multi-trace operators in long multiplets there seems to be a new class of operators that, though not satisfying any known shortening condition, have vanishing anomalous dimensions.

Konishi-like operators decouple both from non-perturbative (instanton) correlators as well as from the strong coupling AdS results but they represent the only available window on genuine string dynamics [35]. The OPE algebra at strong coupling requires the inclusion of multi-trace operators of three kinds. Those dual to multi-particle BPS states, those dual to non BPS-states with gravitational corrections to their binding energy and those dual to non BPS states without mass corrections. A deeper understanding of the last two classes of operators would help clarifying profound issues in the AdS/CFT correspondence such as the string exclusion principle that is expected to take over at finite N [2].

More importantly, by deforming $\mathcal{N} = 4$ SYM it is possible to flow from the superconformal point to phenomenologically more interesting gauge theories with a dynamically generated mass gap [37–40]. Some insight on RG flows can be gained by means of the open-closed string duality. The running of the gauge coupling can be associated to dilaton-like tadpoles [41] much in the same way as chiral anomalies are associated to RR tadpoles [42].

It is a pleasure to thank the organizers of the D.V. Volkov Memorial Conference "Supersymmetry and Quantum Field Theory" in Kharkov, July 2000, for the kind invitation and the pleasant and stimulating atmosphere. I would also like to thank O. De Wolfe, D. Freedman, M. Green, S. Kovacs, J. F. Morales, K. Pilch, G. Rossi, K. Skenderis and Ya. Stanev for enjoyable collaborations on the subject.

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[5]Notice that the coupling constant g_{YM} is related to the one in [29,20] by $2g_{YM} = g$. I thank H. Osborn for pointing out to me the discrepancy with [33].

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